Correspondence

Microwave Optical Ring Resonators

Introduction

Ring resonators have been investigated theoretically and experimentally in the optical region. The ring is formed by three or four mirrors arranged in a triangle or quadrangle, respectively [1]–[6]. Although waveguide-type ring resonators have been known since 1956 [7], little is known about mirror ring resonators in the microwave region.

This report describes properties and applications of quasi-optical ring resonators for the frequencies 17 GHz and 70 GHz.

Theory

The theoretically simplest model is given by three or more mirrors arranged on a circle and with a regular polygon as closed optical path. The mirrors are all identical and have an ellipsoidal reflecting surface (Fig. 1). For optical wavelengths, mirrors of this form are difficult to make but for microwaves ellipsoidal mirrors can be made. One procedure is described in this report, another has been published elsewhere [8]. Ring resonators as mentioned above show no astigmatism, if the reflecting surface of the mirrors represents a section of the equatorial zone of a prolate rotational ellipsoid with the two axes

$$a = 2R \sin (\pi/N),$$
$$b = 2R \sin^2 (\pi/N). \qquad (1)$$

N = Number of mirrors, R = Radius of the tangential circle in the confocal case. The mirror surface can be approximated by a surface which has two different radii of curvature in two perpendicular directions: in the plane of the ring

$$r_0 = 2R$$

and perpendicular to that plane

$$r_\perp = b.$$

Within this approximation the theory of Boyd and Gordon [9] can be extended to ring resonators [10]. According to this theory, the resonant modes are essentially the same as those found for the Fabry-Perot resonator with spherical mirrors. The field of the modes can be calculated using the formulas given in [9]. The distance d of the mirrors in [9] has to be interpreted as the distance d of two adjacent mirrors. The radius of curvature b (confocal case) and b' (nonconfocal case), respectively in [9] has to be replaced by the major axis a of the ellipsoid [see (1)]. In the confocal case a equals d (Fig. 2). Diffraction losses can also be calculated using the formulas by Boyd and Gordon. They calculated the loss as a function of the size of the mirrors which were assumed to be squares having the side $2s$. For the ring resonator one finds the same loss per mirror if the mirrors are assumed to be rectangles with the sides $2s$ and $2s/\sin (\pi/N)$. The Q factor of the resonator caused by diffraction losses is

Manuscript received May 24, 1966; revised August 22, 1966.

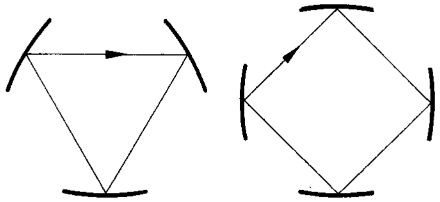

Fig. 1. Ring resonators with three and four mirrors

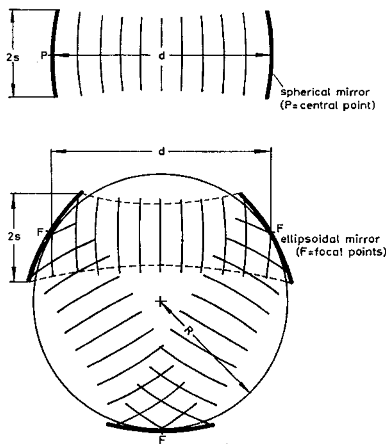

Fig. 2. Comparison of Fabry-Perot resonator and ring resonator. The field pattern between two mirrors each and the diffraction loss per mirror are equal for both resonators if the corresponding values are equal. Radius of curvature (b) of the spherical mirrors (Fabry-Perot) = major axis (a) of the ellipsoid. Distance (d) of the spherical mirrors (d) = distance (d) of two adjacent ellipsoidal mirrors. Size of the spherical mirrors $2s$ by $2s$ corresponds to the size of the ellipsoidal mirrors $2s$ by $2s/\sin (\pi/N)$. The figure shows the confocal case $d = a$.

$$Q_D = \frac{2\pi d}{\lambda(1 - |\sigma_m \sigma_n|^2)}$$

(σ_m, σ_n are eigenvalues of the integral equation; see [9]). In practice mirrors can be made showing small diffraction losses, as compared with ohmic losses. In this case the Q is given by

$$Q_\Omega = \frac{\pi d}{\lambda p},$$

$1-p$ being the voltage reflection coefficient of a plane wave reflected at a plane mirror. For a mode having an electric vector in the ring plane

$$p = \frac{\sqrt{2\omega\epsilon_0/\sigma}}{\sin (\pi/N)}$$

and for an electric vector perpendicular to that plane

$$p = \sqrt{2\omega\epsilon_0/\sigma} \sin (\pi/N).$$

(σ = conductivity of the mirror metal [11]).

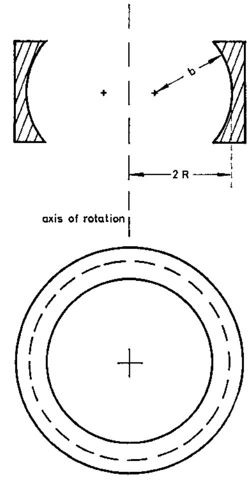

Fig. 3. Ring-shaped body of Al for fabrication of mirrors.

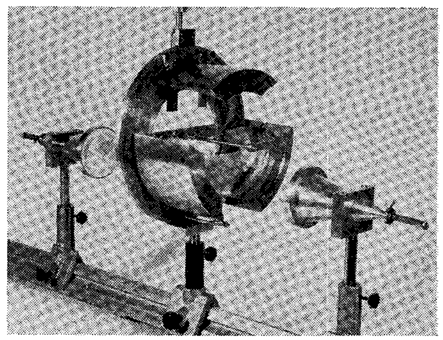

Fig. 4. Ring resonator with two horn antennas placed on an optical bench.

Fabrication of Mirrors

The mirrors were made on a conventional lathe. A ring-shaped body of aluminium was formed on the lathe and later on cut into several pieces, each piece constituting one mirror (Fig. 3). The determining dimensions are the two radii of curvature

$$r_0 = 2R$$
$$r_\perp = b = 3R/2$$

for a three-mirror resonator (N=3). Mirror surfaces made by this procedure are good approximations to the exact ellipsoid.

Two ring resonators have been built, one for the 70 GHz band (R=4. 5 cm, $2s$=9 cm) (Fig. 4) and one for the 17 GHz band (R=10 cm, $2s$=15 cm).

Measurements and Applications

The complete resonator is mounted on an optical bench (Fig. 4). Coupling is accomplished by means of a thin polythene foil which is put up in the middle between the two lower mirrors in such a way that a wave radiated by one of the two horn antennas is reflected towards one of the two mirrors. Tuning can be effected by moving the upper mirror vertically.

As a circuit element a ring resonator of this kind has the property that no wave is reflected back to the source. It is matched at any frequency. Besides this no wave is transmitted at the critical coupling. In this special case all the power is consumed in the ring giving an ideal absorption filter for the resonance frequency [12]. Two or more ring resonators can be placed one after the other to obtain a filter of arbitrary absorption curve. The resulting transmission coefficient is simply the product of the individual transmission coefficient.

At a wavelength of 4 mm the measured loaded Q of the TEM_{00q}-mode at low coupling was 200 000. The theoretical unloaded Q is 250 000 using the dc conductivity of Al and neglecting diffraction losses. At a wavelength of 1.8 cm the corresponding values for the larger ring resonator are: measured loaded $Q=130\,000$ and theoretical $Q_0=200\,000$.

An interesting property of a ring resonator is its degeneracy with respect to the propagation direction of the wave. The wave can circulate clockwise and counterclockwise. Normally only one wave is excited, but if a coupling of the two waves is introduced by placing a dielectric sheet or something else in the path as shown in Fig. 5, standing waves are excited, the resonance splits up, and two resonance frequencies appear (Fig. 6). These two frequencies correspond to different standing wave patterns. In one case the dielectric sheet lies in the plane of a maximum of the electric field (lower frequency), in the other case it lies in a minimum of the electric field (higher frequency).[1] The splitting is a function of the reflection coefficient r of the sheet. The theory gives the relative distance of the two resonance frequencies as

$$\frac{\Delta\omega}{\omega} = \frac{\lambda}{\pi L} \cdot \arctan \sqrt{\frac{r^2 - (\pi L/\lambda Q)^2}{[1 - r^2][1 + (\pi L/\lambda Q)^2]}}, \quad \frac{\pi L}{\lambda Q} \ll 1, \quad r \geq \frac{\pi L}{\lambda Q} \cdot \qquad (2)$$

r = reflection coefficient of the coupling element (sheet),
λ = wavelength,
L = perimeter of the triangle = length of the optical round path,
Q = loaded quality factor of the resonator without the coupling element.

Only if the Q of the resonance is sufficiently high can the two resonances be resolved. This is the case if $r > \pi L/\lambda Q$. At small reflection coefficient and $r \gg \pi L/\lambda Q$ (2) reduces to

$$\frac{\Delta\omega}{\omega} \approx \frac{\lambda r}{\pi L}.$$

The reflection coefficient is transformed directly into the frequency scale and thus can be measured with high precision. By this method permittivities of thin dielectric foils have been measured at 17 GHz (Fig. 6). The smallest r which could be measured is

$$r \approx 0.001$$

[1] The phenomenon may be illustrated by the well-known coupled pendula. Without coupling each pendulum represents one of the two degenerate circulating waves in resonance. If coupled by a spring or the like, the system of the pendula shows two frequencies which are separated by the beat frequency.

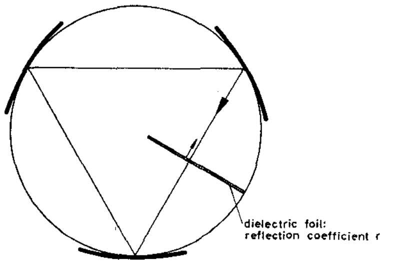

Fig. 5. Coupling of the two degenerate modes by a dielectric sheet with a reflection coefficient r.

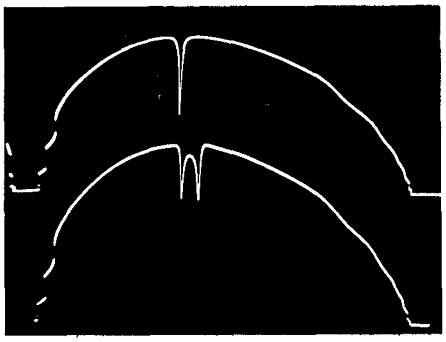

Fig. 6. Transmitted power vs. frequency of the 17 GHz ring resonator. The upper trace shows the resonance curve of the TEM_{00q}-mode. The lower trace shows the effect of a dielectric foil (0.03 mm thick) when placed in the resonator as shown in Fig. 5. The distance of the two resonances is 1.7 MHz.

for the 17 GHz as well as for the 70 GHz band.

Conclusion

Quasi-optical ring resonators for microwaves, millimeterwaves, and submillimeterwaves have interesting properties as circuit elements and measuring instruments. The fact that a ring resonator is always matched and that total absorption at the resonance frequencies is possible, makes it especially suitable for filtering and controlling purposes. Coupling of the two degenerate modes allows the measurement of small reflection cofficients not attainable with conventional methods.

The splitting of the two resonant modes may find an important application in laser technology: a ring laser may be constructed which radiates at two frequencies with a frequency difference adjustable from zero to $c/2L$ [see (2)]. For laser mixing experiments this means that the beat frequency can be varied continuously by introducing a variable coupling.

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Cross Polarization in a General Two Plate Polarizer

A common type of polarizer consists of two quarter-wave plates in a circular TE_{11} mode waveguide. Ideally, the first plate converts linear to circular polarization, and the second rotatable plate converts from circular to linear polarization at an angle $\pi/4$ to this plate. The plates take various forms such as dielectric vanes[1] and disks,[2] or capacitive pins.[3]

This ideal situation does not exist over a finite bandwidth, and perhaps not even at any given frequency; and a cross-polarized output component is present. Values of this cross-polarization for arbitrary vane angles and differential phase shifts would be useful in predicting the performance over a bandwidth of a polarizer using elements of known frequency vs. differential phase shift characteristics.

This correspondence contains generalized curves for the worst cross polarization with rotation for arbitrary values of angular setting of the first vane ϕ_1, and differential phase shifts of θ_1, θ_2 for the first and second vanes, respectively, as shown in Fig. 1. The analysis assumes lossless elements throughout.

Assuming that $E_i = \cos \omega t$ in the direction shown, one finds the output components of E parallel and perpendicular to plate 2 (E_{21}, E_{22}, respectively) to be

$$E_{21} = \cos(T - \theta_1 - \theta_2)\cos\phi_1\cos(\phi_1 - \phi_2) + \cos(T - \theta_2)\sin\phi_1\sin(\phi_1 - \phi_2) \equiv E\cos T + F\sin T$$

Manuscript received April 21, 1966.

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